Linear inequalities

Linear inequalities we are solving similar as equations (see linear equations) with equivalent transformation.

It is important to say that the direction of inequality changes when the whole inequation we multiply with a negative number.

Example:

$2x < 10$

$x < \frac{10}{2}$

$x < 5$

$-2x < 10$ divided with (-2)

$x > \frac{10}{-2}$

$x > -5$

1) Solve the inequalities: $3(x-2)+9x < 2(x+3)+8$

$3(x-2)+9x < 2(x+3)+8$

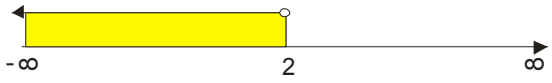
$3x-6+9x < 2x+6+8$

$3x+9x-2x < 6+8+6$

$10x < 20$

$x < \frac{20}{10}$

$\boxed{x < 2}$



$x \in (-\infty, 2)$

2) Solve the inequalities : $\frac{2a+1}{3} - \frac{3a-2}{2} \geq -1$

$\frac{2a+1}{3} - \frac{3a-2}{2} \geq -1$ → entire inequalities multiply 6 (S for 3 and 2)

$2(2a+1) - 3(3a-2) \geq -6$

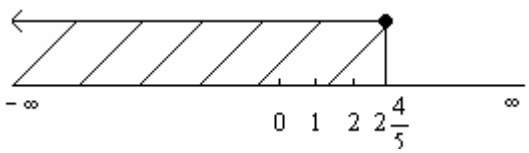
$4a+2-9a+6 \geq -6$

$4a-9a \geq -6-2-6$

$-5a \geq -14$ → man: share with (-5) and the sign turns

$a \leq \frac{-14}{-5}$

$a \leq +2\frac{4}{5}$



In a set of R solutions are: $a \in \left(-\infty, 2\frac{4}{5}\right]$

See: For example,if we seek solutions in the set N (natural numbers), then it was only{1,2}

3) Solve the inequalities: $2x + a > ax - 3$

$2x + a > ax - 3$

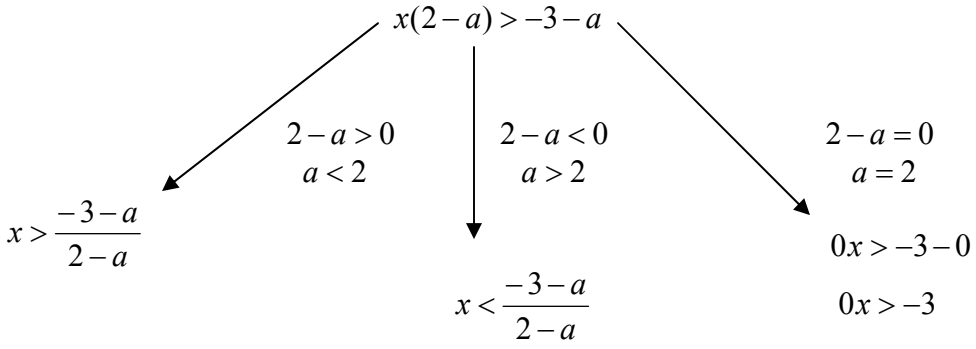
$2x - ax > -3 - a$

$x(2 - a) > -3 - a$

How now?

Is $2 - a$ positive or negative, or maybe zero?

Must withdraw all 3 situation!



For $a < 2 \Rightarrow x \in \left(\frac{-3-a}{2-a}, \infty \right)$

For $a = 2 \Rightarrow x \in R$

For $a > 2 \Rightarrow x \in \left(-\infty, \frac{-3-a}{2-a} \right)$

4) Solve the inequalities:

a) $(x-1) \cdot (x-4) > 0$

b) $(x+3) \cdot (x-5) \leq 0$

In this type of inequalities we use:

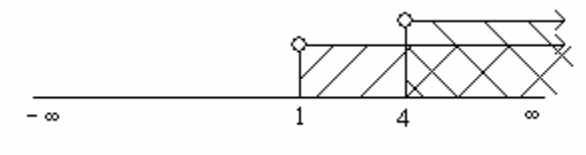
$A \cdot B > 0 \quad \Leftrightarrow \quad (A > 0, B > 0) \quad \vee \quad (A < 0, B < 0)$

$A \cdot B < 0 \quad \Leftrightarrow \quad (A > 0, B < 0) \quad \vee \quad (A < 0, B > 0)$

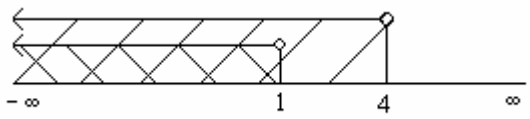
a) $\underbrace{(x-1)}_{A}\cdot\underbrace{(x-4)}_{B}>0$

$(x-1>0, x-4>0) \quad \vee \quad (x-1<0, x-4<0)$

$(x>1, x>4) \quad \vee \quad (x<1, x<4)$



$x \in (4,\infty)$



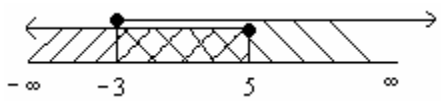
$x \in (-\infty,1)$

Finally solution is : $x \in (-\infty,1) \cup (4,\infty)$

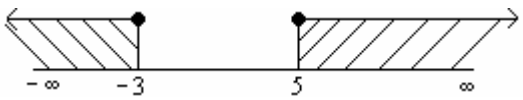
b) $\underbrace{(x+3)}_{A}\cdot\underbrace{(x-5)}_{B}\le 0$

$(x+3\ge 0, x-5\le 0) \quad \vee \quad (x+3\le 0, x-5\ge 0)$

$(x\ge -3, x\le 5) \quad \vee \quad (x\le -3, x\ge 5)$



$x \in [-3,5]$



$\varnothing$ empty set

So, finally, the solution is $x \in [-3,5]$

5) Solve the inequalities: $\dfrac{6-x}{3-x}<-2$

$$\frac{6-x}{3-x}<-2$$

$$\frac{6-x}{3-x}+2<0$$

$$\frac{6-x+2(3-x)}{3-x}<0$$

$$\frac{6-x+6-2x}{3-x}<0$$

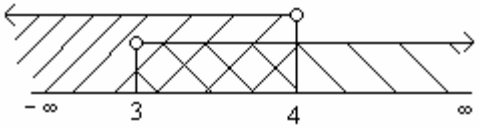
$$\frac{12-3x}{3-x}<0$$

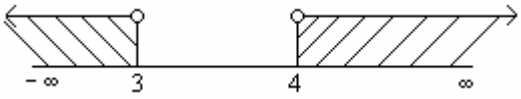
Take heed: To use a"template"on the right side must be zero, and therefore -2 we must move to the left side!

$(12-3x>0 \wedge 3-x<0)$ or $(12-3x<0 \wedge 3-x>0)$

$(-3x>-12 \wedge -x>-3)$ $(-3x<-12 \wedge -x>-3)$

$(x<4, x>3)$ or $(x>4, x<3)$





$x \in [3,4]$ finally solution

$\varnothing$

6) Solve the inequalities: $-3 < \frac{n-1}{n+1} < 5$

Here we must solve 2 inequalities, and than "group" their solutions.

$$-3 < \frac{n-1}{n+1} < 5$$

$-3 < \frac{n-1}{n+1}$ then: $0 < \frac{n-1}{n+1} + 3$

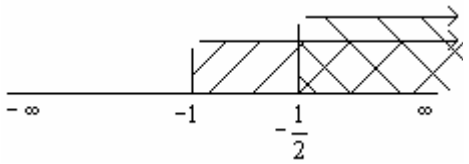
$$0 < \frac{n-1+3n+3}{n+1}$$

$$0 < \frac{4n+2}{n+1}$$

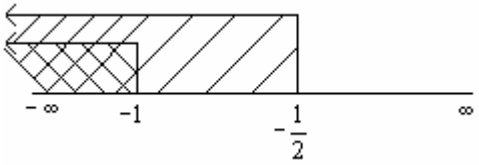
So: $\frac{4n+2}{n+1} > 0$

$(4n+2>0 \wedge n+1>0) \vee (4n+2<0 \wedge n+1<0)$

$(n > -\frac{1}{2} \wedge n > -1) \vee (n < -\frac{1}{2} \wedge n < -1)$



$n \in \left(-\frac{1}{2}, \infty\right)$



$n \in (-\infty, -1)$

For the first part of the task , the solution is: $n \in (-\infty, -1) \cup \left(-\frac{1}{2}, \infty\right)$

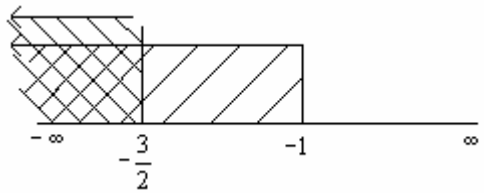
Second part:

$$\frac{n-1}{n+1}<5 \quad \Rightarrow \quad \frac{n-1}{n+1}-5<0 \quad \Rightarrow \quad \frac{n-1-5n-5}{n+1}<0$$

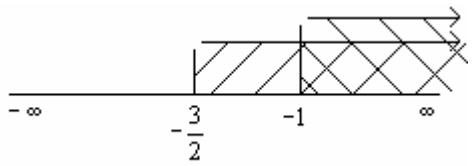
So: $\frac{-4n-6}{n+1}<0$

$$(-4n-6>0 \wedge n+1<0) \quad \vee \quad (-4n-6<0 \wedge n+1>0)$$

$$(n<-\frac{3}{2} \wedge n<-1) \quad \vee \quad (n>-\frac{3}{2} \wedge n>-1)$$



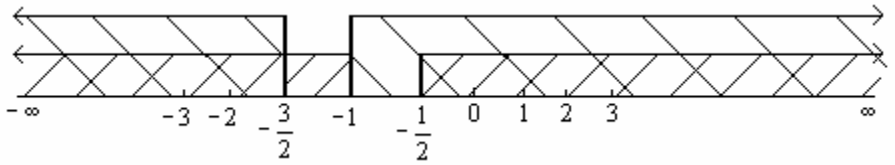
$n \in \left(-\infty,-\frac{3}{2}\right)$



$n \in (-1,\infty)$

For the first part of the task , the solution is: $n \in \left(-\infty,-\frac{3}{2}\right) \cup (-1,\infty)$

Together:



Finally solution is:

$$\boxed{n \in \left(-\infty,-\frac{3}{2}\right) \cup \left(-\frac{1}{2},\infty\right)}$$